ASSESSING STUDENTS' UNDERSTANDING OF THE CONCEPT OF DIFFERENTIATION AND A FUNCTION'S PARAMETERS

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In order to assess students' understanding of the concept of differentiation and the meaning of parameters of a function (like an intercept), several test items were specifically constructed as well as gathered from other studies. The items were piloted and then administered to 2665 upper secondary school students in in the German federal state of North Rhine-Westphalia. The Rasch model was used to scale the results on an interval level. Our results show significant gender differences and let us judge the difficulty of certain mathematical tasks.

INTRODUCTION

In order to measure and get a deeper insight into student's conceptual knowledge in the field of functional relationships and differentiation an achievement test was elaborated. The test focuses on student's images of differentiation as well as the meaning of the parameters of a function, e.g. the addition of a constant. These contents have to be compulsorily taught until the end of the first senior class (ca. age 16) in the German federal state of North Rhine-Westphalia (so-called "*Einführungsphase der gymnasialen Oberstufe"),* so that we administered our test at the end of those classes.

The dichotomous Rasch-model is used to measure the students' achievement on an interval scale.

The test was primarily designed to be used in a research project measuring the student's learning outcomes of a Continuous Professional Development program for teachers (cf. Thurm et al. 2015). However, the developed instrument is kept general enough to be used in other projects.

BACKGROUND

We will shortly outline what we mean by 'conceptual understanding' or synonymously 'understanding of a concept'. Tall & Vinner (1981, p. 152) use the term 'concept image' to describe "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes". To be more specific, part of a students' concept image of differentiation can (and ideally should) have different facets: One can focus the local linear best approximation property of a derivative or see it as a local rate of change. Another student might concentrate more on the derivative being the slope of the graph's tangent in a certain point (e.g. Hußmann & Prediger 2010). Some activities or tasks need at least one of those foci as part of a student's concept image. E.g., the graphical construction of an only visually given function's derivative graph demands the student to see the derivative as the slope of the graph's tangent which varies for each point.

The same holds for students' concept image of functions in general (e.g. Vinner 1983).

AIM OF THE STUDY

On the one hand, our research aims on providing a tool to measure students' understanding of the concept of differentiation and a function's parameters. On the other hand, we try to evaluate to which degree students' abilities and conceptual understanding is developed at the end of year 10. Thus, we focus on the genesis of a testing instrument as well as on a summative assessment.

DEVELOPING THE INSTRUMENT

First of all, a large set of items was elaborated. On the one hand we designed items ourselves, on the other hand we took items from literature (e.g. the TIMS study, e.g. Baumert et al. 2000). The item set was piloted before the administration of the main study took place. Items which did not work as intended (e.g. if the validity could not be ensured) were redesigned or dismissed. Moreover, experts were involved in the process of item design and selection.

To guarantee person independence whilst maximizing test efficiency at the same time two test booklets were generated. Both booklets only differ in their items order, not the items themselves. It was ensured that the item difficulty is not booklet dependent.

To scale the assessment data on an interval level the dichotomous Rasch model is used. All item and person parameters are estimated by the software "ACER ConQuest 4". For each person a set of five plausible values is drawn. Statistics we are interested in (like the mean of ability) are computed separately for each plausible value and then averaged. This procedure is analogous to the analysis of PISA data and we refer to the technical report of Adams & Wu (2002) for further details. Plausible values should only be used to describe population characteristics and not individual person properties. In the former case they provide a better estimation than other methods, since those tend to either over- or underestimate the variance (Adams & Wu, p. 255).

As criteria for the goodness of fit so-called mean-square fit statistics (MNSQ) can be used, which exist in form of a weighted and a stricter unweighted variant. The optimal value for both criteria is 1. However, the observed values should not extend 1.2 nor fall below 0.8 for high stakes multiple choice tests (Bond & Fox 2015). Linacre (2002) calls values between 0.5 and 1.5 "productive for measurement".

SURVEY AND RESULTS

The final test comprises 21 items. It was administered to 2665 students (1340 male, 1304 female, 21 unknown gender) between April and May 2015 in the German federal state of North Rhine-Westphalia. Since our study is non-compulsive we can't guarantee a representative sample.

As one can see in table 1, most items fit the model very well ($0.8 < F < 1.2$). However, some show a weak over- or underfit using those boundaries. All are "productive for measurement".

Tall (1993, p. 17) already noticed the "consequent student preference for procedural methods rather than conceptual understanding". We can support this observation to a certain degree since some of the easiest test items (item 1–3) are of more procedural nature. Furthermore, the data allows to estimate the difficulty of certain mathematical tasks. E.g., one can see that students find the translation of a function in x-direction a lot harder than in y-direction, which is indeed consistent to our experience.

I	δ	P	F	T	Task to solve the item
1	-2.27	87.6	1.02	OP	Calculate the derivative of a polynomial with degree 3
2	-1.66	80.6	1.03	OP	Calculate the derivative of a constant function
3	-0.58	63.0	1.01	OP	Calculate the derivative of a monomial with degree n
4	-1.30	74.7	1.17	SC	Identify the appearance of a curved graph when displaying it in an infinitesimal interval
5	-1.20	73.3	0.96	SC	Graphically identify the graph of the derivative
6	1.52	23.7	1.00	OP	Change a function’s term s.t. its graph translates in x-direction
7	-1,37	76.7	1.04	OP	Change a function’s term s.t. its graph translates in y-direction
8	-0.97	69.4	1.02	SC	Identify the average rate of change
9	-0.88	67.6	1.03	SC	Identify the instantaneous rate of change at a certain point
10	0.77	35.5	1.03	SC	Identify the instantaneous rate of change at a certain point
11	0.35	43.6	1.23	SC	Identify the new graph when the function is scaled by a positive factor
12	-2.20	86.9	1.06	SC	Decide whether the derivative of a function is an upwardly or downwardly opened parabola
13	1.09	37.0	1.03	OP	Translate a polynomial of degree 3 s.t. it has exactly two roots
14	1.91	18.4	0.77	MC	Decide whether given properties of a given graph are true for its derivative
15	1.37	25.8	0.84	MC	See item 14
16	0.69	37.4	1.10	MC	See item 14
17	2.13	15.7	0.78	MC	See item 14
18	1.72	20.8	0.77	MC	See item 14
19	-0.30	56.5	0.95	SC	Assign three derivatives’ graphs to their primitives’ graphs
20	0.76	35.7	1.02	SC	Identify which translations of a function’s graph are also inherited by its derivative’s graph
21	0.43	43.6	0.93	OP	Graphically construct the graph of the derivative

Table 1: Overview of the applied items (I), their according difficulty (δ, higher is more difficult), number of correct solutions in percent (P), their weighted MNSQ fit (F), the item’s type (T, SC = single choice, MC = multiple choice, OP = open), and the task

Table 2 shows the average test difficulty in terms of Rasch person estimates and percentage. Note that all values which involve δ are averages over the five statistics for each set of plausible values. The gender differences are significant with $p < 0.001$. Cohen’s d for the gender effect size is 0.30 and 0.32 for Rasch person estimates and percentage, respectively, where positive values imply an advantage of boys. Concerning this matter, our test corresponds to the current state of research (e.g. Hyde et al. 1990). Moreover, M(P) shows that the test is neither too hard nor too easy.

Group	n	M(δ)	M(P)	SD(δ)	SD(P)
all	2665	0.040	50.56	1.07	0.19
male	1340	0.197	53.59	1.11	0.20
female	1304	-0.119	47.48	1.00	0.18

Table 2: Overview of gender differences (n = number of cases, M = arithmetic mean, SD = standard deviation)

CONCLUSION

It was possible to generate a test which fits the Rasch model well. However, it shows sensitivity to gender in the sense that boys show significantly better test results. Moreover, we showed an example how the data can be used to estimate the difficulty of certain mathematical tasks. We will carry out further research in this direction.

REMARKS

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